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MEASUREMENT OF MOMENTS OF INERTIA AND PRINCIPAL INERTIA AXIS OF--ETC(U)

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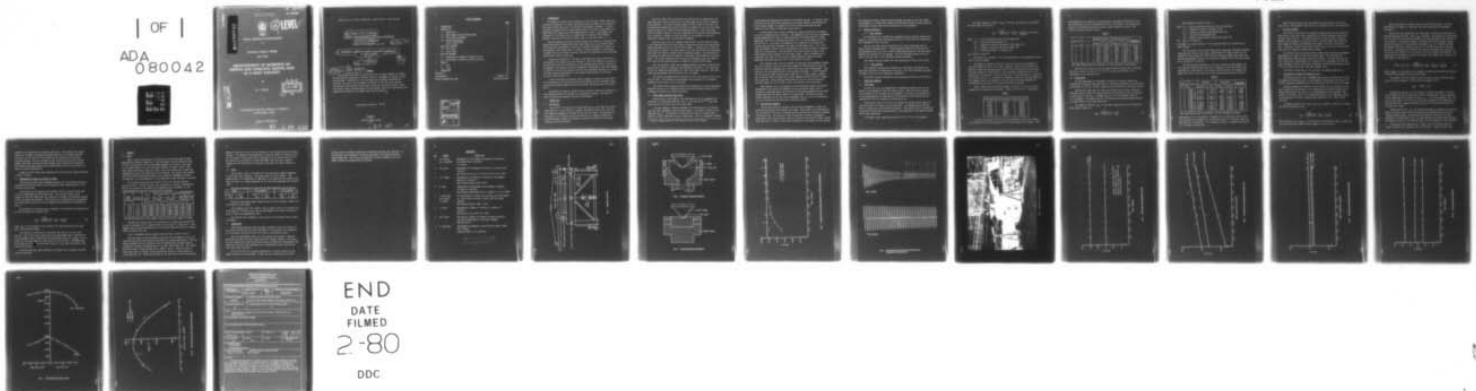
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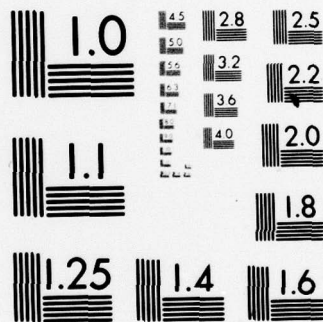
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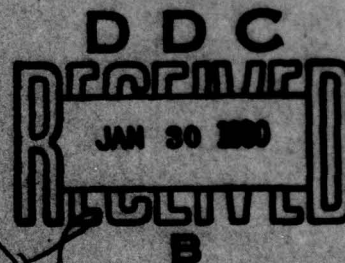
July 1979

MEASUREMENT OF MOMENTS OF  
INERTIA AND PRINCIPAL INERTIA AXIS  
OF A GNAT AIRCRAFT

by

R.L. Poulter

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SUMMARY

The modifications made to a ground rig used to measure moments of inertia, and the technique developed to minimise errors in the moment of inertia in roll, are described. Calibration of the rig shows that acceptable accuracies are obtained, and results for the moments of inertia in roll and pitch, and the inclination of the principal inertia axis of the Gnat aircraft are given. Three fuel states, empty, external tanks full and internal tanks full, were tested, and comparisons are made with estimated values where possible.

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## 1 INTRODUCTION

The moments of inertia and inclination of the principal inertia axis of an aircraft are required to be known particularly when the values of the stability and control derivatives are to be extracted from dynamic responses. For most aircraft, estimated inertial characteristics have to be used, even though it is known that the estimates are often significantly in error. For example, the estimates for the Fairey Delta 2, Handley Page 115 and Avro 707B differed markedly from results obtained on ground rigs, as reported in Refs 1 to 4.

Some discrepancies had been observed between the results for stability and control derivatives obtained from tunnel and flight tests on the Gnat<sup>5</sup>, which could have been attributable to the use of inaccurate estimates of inertias in the analysis of the flight data. In order to clarify matters, it was decided to measure the roll and pitch inertias, and also the inclination of the principal inertia axis, using a ground rig.

During the calibration of the rig, it was found that some alterations to the design of the original rig were advisable. These are described in section 2, and the results for the calibrations of the modified rig are given in section 3. A technique to determine the various components contributing to the total moments of inertia of the rig and aircraft also had to be developed (section 4), in order to derive accurate measurements of the inertia characteristics of the Gnat aircraft itself (section 5).

It was found that the measured value of moment of inertia in pitch and of the inclination of the principal inertia axis agreed with the estimated values, but that the measured moment of inertia was about 30% greater than that used in Ref 5.

Throughout this Report, Imperial units are used to derive the inertias as the weighing machines and weights were calibrated in pounds. The tables of results display both Imperial and metric equivalents.

## 2 INERTIA RIG

### 2.1 Description

The rig (Fig 1) consists of a large rigid base, a roll frame and a pitch frame which carries two cradles to support the aircraft under test. The rig was originally designed for the BAC 221 but was never used. The principle of the system is to oscillate the aircraft and rig as a compound pendulum in a controlled manner, measure the period of oscillation and derive the inertia knowing the spring stiffness.

The roll frame, which incorporates the pitch frame, is supported on the rigid base and is free to pivot about two knife edges in the plane of symmetry of the aircraft. Two sets of springs connect the roll frame to the base, and are positioned symmetrically about the plane of symmetry so that the roll frame oscillates when it is released from a displaced position. The pitch frame, which is rigidly attached to the roll frame when roll inertia is being measured, can be adjusted to change the pitch attitude of the aircraft so that roll inertias at varying pitch attitudes may be measured.

When the rig is set up to measure pitch inertia the roll frame is fixed to the rigid base while the pitch frame is freed and can then pivot about knife edges positioned just aft of the CG of the system. A single set of springs at an aft position connects the pitch frame to the base and provides the restoring force when the frame is released from a displaced position. The tensioning of the springs is achieved by the moment of the weight acting forward of the knife edges. The cradles supporting the aircraft are arranged so that the combined CG of the aircraft and pitch frame is forward of the pitch knife edges.

The roll and pitch frames were weighed separately and their respective CG positions were found by experiment and checked by calculation. The total mass of the roll frame was approximately 2/3 that of the empty aircraft. The pitch frame was approximately half the mass of the roll frame.

The rig was instrumented with sensitive ( $\pm 10^\circ/\text{s}$ ) rate gyros in both pitch and roll axes. The signals from these gyros were recorded on a UV recorder running at 4 in/s with a 1/10 s time base.

The release from a displaced position was achieved by a release mechanism mounted on the top of a standard aircraft jack, the displacement being set by extending the jack to the required position.

## 2.2 Knife edges and knife edge blocks

Historically, at RAE Bedford, the knife-edge and block arrangement has consisted of a knife edge which fits into a vee block (Fig 2)<sup>1-3,6</sup>, the idea being that the vee block centres the knife edges.

It became apparent, during preliminary roll calibrations of the inertia rig, that this arrangement gave an unusual shape to the graph of period versus amplitude (Fig 4). The period\* of oscillation was found to be appreciably shorter for small amplitudes than for larger ones. This was confirmed by making

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\* Period = average period of the first ten cycles after release.

a long record and measuring the periods as the motion decayed. It was also found that when the vee blocks were replaced by flat plates (Fig 3), and nothing else in the system changed, the graph of roll period versus amplitude was much flatter over the range of amplitudes tested (Fig 4).

It also became obvious when recording the longer records that the damping of the two systems was very different. The damping with the vee block arrangement started comparatively large, at large amplitudes, and became smaller at small amplitudes until below a limiting amplitude,  $0.2^{\circ}$ , it was the same as for flat plates (Fig 5). In contrast the damping of the system with flat plates was constant and very small throughout the range of amplitudes, Fig 5. For a  $0.8^{\circ}$  input the number of cycles to half amplitude for vee blocks was about 9 compared with 101 for flat plates. This indicates that there is more friction at the knife edge/vee block interface at the larger inputs. This is probably due to scuffing on the sides of the block for larger amplitudes whereas for small amplitudes the knife edge just rolls in the bottom of the vee.

This scuffing would also be increased if there was any misalignment between the two vee blocks. The vee blocks were capable of being rotated to align the vees, but this only happened under load if there was a gross misalignment. This was due to the increased friction at the ball/cup interface. It was noticeable that in pitch the same tendencies were shown but to a lesser degree, probably due to the fact that the blocks were closer together and hence could be aligned more accurately. This aligning problem does not occur in the flat plates as the knife edges are free to take up their own position across the plates.

There appears to be a discrepancy, as yet unresolved, between the effects of amplitude on the period and damping with the vee blocks (Figs 4 and 5). Friction at large amplitudes would account for the higher damping but this seems inconsistent with the shorter period at low amplitude. The more consistent results obtained with the flat plate arrangement led us to use it in preference to the vee blocks that had been used in previous studies.

### 2.3 Rollspring attachment

The rig was originally designed with a see-saw arrangement to apply the motion of the spring to the movable roll frame. This proved to be unsatisfactory as the friction at the pivot increased when the springs were tensioned so damping the oscillations. The function of this see-saw was to provide a mechanical advantage so that large aircraft could be oscillated with relatively small springs. Since the Gnat is a small aircraft this mechanism was unnecessary and was deleted,

the springs then being connected directly between the base and the roll frame. An arrangement of crossed knife edges<sup>3</sup> was positioned between the base frame and the springs to remove some of the constraints at the attachment.

## 2.4 Spring calibration

### 2.4.1 Roll springs

The individual roll springs were calibrated using a Dennison tensile test machine. The springs were then matched into sets so that the total stiffness of each comparable set was the same.

The sets were then check calibrated in the rig by tensioning them by pulley from an overhead gantry. A spring balance between the pulley and the spring measured the force, and a vernier height-gauge mounted on the base frame measured the extension of the springs. The total stiffness proved to be the same as the sum total of the individual spring stiffnesses.

The nominal spring stiffness for each spring was 75 lb/in (13134.5 N/m).

### 2.4.2 Pitch springs

The individual springs were calibrated on the tensile test machine and were check calibrated in combination on the rig. The total stiffness was again the same as the sum of the individual stiffnesses.

The nominal spring stiffness for each spring was 100 lb/in (17512.7 N/m).

## 3 BASIC RIG INERTIAS

### 3.1 Roll frame

For the roll rig, two combinations of springs were tested, one set consisted of two springs per side, the other of four. The sets of four springs were intended for the aircraft and rig combined. The sets of two springs were to give approximately the same period of oscillation to the rig alone as was expected with the four springs for aircraft plus rig.

Records were taken for both sets of springs at different pitch angles of the pitch frame over ranges of input amplitudes. The average period over the first 10 cycles for each condition was plotted. The period used for calculating the inertia was derived by extrapolating to zero amplitude on the graph of period versus input amplitude<sup>1</sup>.

The range of input amplitudes was from  $0.2^{\circ}$  to  $2^{\circ}$  in  $0.2^{\circ}$  increments.

The total moment of inertia,  $A_{KE}$  of the roll rig about an axis through the knife edges is given by:-

$$A_{KE} = \left( \frac{P_x}{2\pi} \right)^2 (\lambda_x y_s^2 - W_R z_R) \quad \text{(equation (1) derived in Ref 1)}$$

where  $P_x$  = roll oscillation period (at zero amplitude)  
 $W_R$  = total oscillating weight (lb)  
 $y_s$  = distance of springs outboard of knife edges (ft)  
 $z_R$  = height of system CG above knife edges (ft)  
 $\lambda_x$  = combined stiffness of springs lb/ft

$A_{KE}$  being in slugs ft<sup>2</sup>.

The weight  $W_R$  includes 1/3 of the spring weight as an approximate correction for heavy springs<sup>7</sup>.

The moment of inertia of the roll rig plus two sets of two springs at zero pitch angle was found to be 1034.5 slugs ft<sup>2</sup> (1402.5 kg m<sup>2</sup>) whereas the moment of inertia of the rig plus two sets of four springs at zero pitch angle was 1066.2 slugs ft<sup>2</sup> (1445.6 kg m<sup>2</sup>). The calculated contribution due to the springs was 12.0 slugs ft<sup>2</sup> (16.3 kg m<sup>2</sup>) for the 2 x 2 springs and 24.1 slugs ft<sup>2</sup> (32.6 kg m<sup>2</sup>) for the 2 x 4 springs the difference being less than that observed. This suggests that it is important to use the actual spring arrangement for calibrating the rig that is to be used with rig plus aircraft.

Table 1 below shows the derived moments of inertia of the roll rig and springs at the various pitch attitudes. These are for the 2 x 4 spring sets.

Table 1

Pitch angle	Inertia slugs ft <sup>2</sup>	Inertia kg m <sup>2</sup>
+7.56	1119.9	1518.3
+4.56	1089.2	1476.7
+3.06	1080.5	1465.0
+1.56	1079.6	1463.8
0.00	1066.2	1445.6
-1.48	1058.2	1434.8
-3.00	1049.3	1422.6
-4.50	1049.6	1423.0

To test the accuracy of the roll rig measurements, weights were added to the rig and the tests were repeated with the pitch frame at zero angle. This

gave figures for the inertia of rig plus weights, from which the inertia of the rig was deducted to give the measured inertia of the weights. The weights were placed in two different positions to give a wider spread of inertias. The results are given on Table 2 below:

Table 2

Position	No. of wt	Measured inertia		Calculated inertia		$\frac{\text{Measured}}{\text{Calculated}}$
		slugs ft <sup>2</sup>	kg m <sup>2</sup>	slugs ft <sup>2</sup>	kg m <sup>2</sup>	
1	2	41.7	56.6	44.3	60.1	0.941
1	4	81.3	110.3	82.5	111.8	0.986
1	6	123.4	167.3	120.6	163.6	1.023
1	8	158.7	215.1	159.1	215.8	0.997
1	9	172.7	234.1	178.7	242.2	0.967
2	2	93.7	127.0	93.4	126.6	1.003
2	4	184.5	250.2	180.9	245.2	1.020
2	6	267.3	362.3	267.1	362.1	1.000
2	8	351.5	476.5	354.7	480.9	0.991

NB These inertias are all referred to an axis through the knife edges.

The rms of the errors (*ie* the discrepancy between measured and calculated inertias) is about 2.9 slug ft<sup>2</sup> (3.9 kg m<sup>2</sup>), and the error appears to be independent of the absolute value.

### 3.2 Pitch frame

For the measurement of the moment of inertia of the pitch frame, two combinations of springs were again used, to allow more flexibility of positioning the aircraft on the rig. One set consisted of two springs and the other of three.

Records were taken when the frame was released from an offset condition. Different sizes of inputs were again used but the variation was not as large due to constraints on the rig. A maximum value of 1.8° was achieved, again in 0.2° increments. The period used for calculating the inertia was again derived by extrapolating to zero input amplitude.

The moment of inertia  $B_{KE}$  for the pitch frame about an axis through the knife edge is given by:

$$B_{KE} = \left( \frac{P}{2\pi} \right)^2 \left( \lambda_y X_s^2 - W_T Z_T \right) \quad (2)$$

This equation is derived in Ref 1,

where  $P_y$  = pitch oscillation period (s) (at zero amplitude)  
 $\lambda_y$  = combined stiffness of springs (lb/ft)  
 $X_s$  = distance of springs behind knife edge (ft)  
 $W_T$  = total oscillating weight (lb)  
 $Z_T$  = height of pitch frame CG above knife edges (ft)  
 and  $R_{KE}$  is in slugs ft<sup>2</sup>.

The weight  $W_T$  again includes 1/3 of the spring weight as a correction for heavy springs.

The two results for the pitch frame inertia using the two spring configurations were very close to each other, 1406.0 slugs ft<sup>2</sup> (1906.2 kg m<sup>2</sup>) for two springs and 1415.3 slugs ft<sup>2</sup> (1918.9 kg m<sup>2</sup>) for three springs, well within the expected experimental scatter; the calculated difference due to the extra spring was 5.5 slugs ft<sup>2</sup> (7.5 kg m<sup>2</sup>).

The rig was then ballasted by a bar and a variety of lead weights, and the tests were repeated using both sets of springs to check the accuracy and repeatability of the system. The inertias thus derived for the weights were again very close to each other and to the calculated inertias (see Table 3).

Table 3

No. of springs	Weights	Measured		Calculated		<u>Measured</u> <u>Calculated</u>
		slugs ft <sup>2</sup>	kg m <sup>2</sup>	slugs ft <sup>2</sup>	kg m <sup>2</sup>	
2	Bar	47.1	63.9	49.2	66.7	0.958
2	Bar +2	381.9	517.8	386.9	524.6	0.987
2	Bar +4	716.2	971.0	725.6	983.8	0.987
2	Bar +6	-	-	-	-	-
2	Bar +8	1390.4	1885.1	1403.3	1902.6	0.991
2	Bar +9	1554.0	2106.9	1574.4	2134.5	0.987
3	Bar	-	-	49.2	66.7	-
3	Bar +2	386.5	524.1	388.0	526.1	0.996
3	Bar +4	716.2	971.0	726.7	985.3	0.985
3	Bar +6	1050.3	1424.1	1064.4	1443.2	0.987
3	Bar +8	1381.3	1872.8	1403.3	1902.6	0.984
3	Bar +9	1572.5	2132.0	1574.4	2134.5	0.999

NB These inertias are all referred to an axis through knife edges.

Unfortunately it is not possible to compare all the conditions with two and three springs as the weights were inadvertently used in a different order and they were not all identical. It is possible, however, to compare directly the 8 or 9 weight configurations.

These figures indicate that the inertias are within about 1.5% for the large values. The measured values here are all smaller than the calculated ones.

#### 4 RIG PLUS AIRCRAFT

The aircraft was placed in the trestles mounted upon the pitch frame and bolted down so that there was no possibility of relative movement between the aircraft and rig (Fig 6). The centre of gravity of the aircraft, previously measured, was arranged to be vertically above the knife edges about which the pitch frame was rotated for different pitch angles in the roll oscillation case. This was to ensure the vertical distance ( $Z_{AC}$ ) of the CG of the aircraft above the knife edges was substantially constant.

The aircraft was fitted with external slipper tanks for these experiments. Measurements were taken at three fuel states: empty, full internals with empty externals, empty internals with full externals. For both the pitch and roll oscillations, displacements up to maxima of  $2^\circ$  at increments of  $0.2^\circ$  were used, although at some of the heavier conditions the minimum displacements were  $0.8^\circ$  as the smaller ones gave very small angular rates.

It should be noted that for all the tests the undercarriage was up.

##### 4.1 Measurement of moment of inertia in roll

The spring and knife edge arrangement means that the aircraft and rig are constrained to oscillate about an axis through the knife edges parallel to the aircraft fuselage longitudinal axis when the pitch frame is at zero pitch angle.

The basic method for obtaining the roll inertia of the aircraft plus rig was the same as for rig alone, i.e. the combination was displaced through small angles and released, the periods of the ensuing motion being measured. The period of the motion used is the hypothetical period at zero amplitude obtained by extrapolating the graph of period versus amplitude, Fig 7, back to zero amplitude.

The basic equation for aircraft plus rig is similar to that for rig alone, but with one additional term,

$$A_{KE} = \left( \frac{P}{2\pi} \right)^2 \left( \lambda_x y_s^2 - W_R Z_R - W_{AC} Z_{AC} \right) \quad (3)$$

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where the extra term  $W_{AC} Z_{AC}$  is the product of the aircraft weight in pounds and the vertical distance above the knife edges of the CG in feet.

Since the aircraft is heavier than the rig and its CG farther from the supporting knife edges it is important that this mass moment be determined accurately.

The aircraft was weighed and the longitudinal CG positions found before it was positioned upon the rig. Hence  $W_{AC}$  was known. The position of the vertical CG of an aircraft is notoriously difficult to obtain and results are usually not very accurate. Conventional attempts to measure the vertical CG gave a large scatter of results so it was decided to determine  $Z_{AC}$  by an indirect method. The weights, which had previously been used in the rig alone case to test the accuracy of the system, and for which the inertias were known, were refitted to the rig with the aircraft and the period of the total assembly was measured. The expression for the moment of inertia of aircraft plus weights plus rig about the knife edges is then given by:

$$MI(AC + W_T + R_{IG}) = \left( \frac{P^2}{2\pi} \right) \left( \lambda_x y_s^2 - W_R Z_R - W_{WT} Z_{WT} - W_{AC} Z_{AC} \right) \quad (4)$$

where  $W_{WT} Z_{WT}$  is the product of the weights in pounds and the vertical distance of the CG of the weights above the knife edges.

The weights were then removed and the rig plus aircraft alone were oscillated, so that the moment of inertia given by equation (3) is

$$A_{KE} = MI(R_{IG} + A_C) .$$

Subtracting equation (3) from equation (4) leaves the moment of inertia of the weights on the left-hand side, which is already known, and an expression on the right-hand side in which the term  $Z_{AC}$  is the only unknown, thus  $Z_{AC}$  could be calculated. The CG of the aircraft was measured for the empty case only and the firm's weight information was used to calculate CG positions for the other fuel states.

Using this value of  $Z_{AC}$  and substituting back into equation (3) gave the moment of inertia of the total aircraft-plus-rig combination about the axis through the knife edges. By subtracting the moment of inertia of the rig, obtained on the preliminary tests, and then applying the parallel axes theorem, the moment of inertia of the aircraft about an axis through the CG was obtained.

The inertias were measured over a range of pitch attitudes with fuel states of empty, full internals, and full externals. Sensible results were

achieved in the empty and full external cases only. The internal fuel system consists of a number of irregularly shaped tanks and it is thought that the vertical CG of the aircraft in the nominally full condition was not known accurately enough. Also any airspaces in the system would allow fuel to move between tanks when the pitch attitude changed, thus altering the CG position. The external tanks being regular and filled through a hole at the top were not subject to these uncertainties.

Graphs of period versus input amplitude for the three fuel states are shown in Figs 7, 8 and 9.

#### 4.2 Measurement of moment of inertia in pitch

The spring and knife edge arrangement ensures that the aircraft and rig is constrained to oscillate about an axis through the knife edges parallel to the aircraft pitching axis.

The system used for measuring the aircraft inertia in pitch was the same as that for measuring the pitch frame alone. The rig and aircraft, while balanced on the knife edges under the influence of springs, was displaced through a series of angles and released. The period of the ensuing motions was measured and a hypothetical period for the zero amplitude was obtained by extrapolation, Fig 10.

The equation for the pitch case is similar to the previous one (equation (2)) but with the addition of another term.

$$B_{KE} = \left(\frac{P}{2\pi}\right)^2 \left(\lambda_y x_s^2 - W_R Z_R - W_{AC} Z'_{AC}\right) \quad (5)$$

where  $Z'_{AC}$  is the distance of the aircraft's CG, derived from the roll case, above the pitch knife edges.

This equation is much better conditioned than that for the roll case. Here the  $\lambda_y x_s^2$  term is much larger than the corresponding term in equation (4) as the springs are much farther from the knife edges, while the  $W_{AC} Z'_{AC}$  term is smaller as the knife edges are nearer the CG of the aircraft. This means that the difference is large and hence small errors in  $Z'_{AC}$  have less effect on the value of the inertia.

Pitch inertias were again measured in the empty, full internals, and full external conditions.

## 5 RESULTS

### 5.1 Roll

Figs 7, 8 and 9 show plots of the periods of roll motion against input amplitude for different conditions. From the graphs it can be seen that there is very little scatter and the relationship appears to be linear. The slope of the graphs appear to be less than those obtained in earlier tests<sup>1</sup>. With tanks empty and external tanks full (Figs 7 and 9) the slopes are so small that extrapolation back to zero amplitude is well defined. The results with internal tanks full (Fig 8) show greater scatter, and a definite dependence of period on amplitude, so the extrapolation to zero amplitude is more difficult to justify.

The following table gives the results of the roll inertias achieved during the tests. The full internal fuel conditions are shown for completeness but are of no value as the answers are incompatible due, presumably, to the unknown position of the fuel at various pitch attitudes.

Frame pitch attitude	Aircraft attitude	Empty		Full internal fuel		Full external fuel		Contribution due to external fuel	
		slugs ft <sup>2</sup>	kg m <sup>2</sup>	slugs ft <sup>2</sup>	kg m <sup>2</sup>	slugs ft <sup>2</sup>	kg m <sup>2</sup>	slugs ft <sup>2</sup>	kg m <sup>2</sup>
4.56	4.16	1946	2638	1942	2633	3167	4294	1221	1655
3.06	2.66	2054	2785	2495	3383	3312	4490	1258	1706
1.56	1.16	2119	2873	3009	4080	3364	4561	1245	1688
0.0	-0.40	2183	2960	3191	4326	3380	4583	1197	1623
-1.48	-1.88	2101	2849	3246	4401	3363	4560	1262	1711
-3.0	-3.40	2095	2840	2572	3487	3334	4520	1239	1680

NB The aircraft sat on the cradles 24 min nose down with respect to frame.

Treating the external fuel as a solid mass at a distance from the aircraft CG gives an estimate for the added inertia of 1235 slugs ft<sup>2</sup> (1674 kg m<sup>2</sup>). This suggests that the inertias measured for the aircraft empty and with full external tanks are within about  $\pm 2\%$ .

A plot of measured inertia against pitch attitude is shown in Fig 11.

Over the range of pitch attitudes tested here the contribution to the roll inertia of the fuel in the external tanks would not change significantly, thus the results obtained with full externals could be combined with the empty case by allowing for the mean contribution due to the fuel (1238 slugs ft<sup>2</sup>). This simple device was used to obtain another set of 'empty' results. These two sets of values were used to calculate a mean second-order curve as a least squares fit to the data (Fig 12). This least squares fit was then used to find the principal

moment of inertia in roll and the inclination of the inertia axis for the empty aircraft. The results thus achieved give a principal moment of inertia in roll of 2157 slugs  $\text{ft}^2$  (2925  $\text{kg m}^2$ ) and an inclination of the inertia axis of  $0.7^\circ$ . The results for the inertia are about 30% higher than the firm's original estimate but the inclination of the axis is very near the firm's estimate of  $0.8^\circ$ .

## 5.2 Pitch

Fig 10 shows a plot of the period of the pitch motion against amplitude for the three fuel conditions. Here again there is little scatter. The graphs appear to be independent of amplitude in the empty and full external fuel cases but there is a slight dependence in the case with supposedly full internal tanks.

The inertias obtained in pitch for the three conditions are as follows:

Empty		Full internal		Full external	
slugs $\text{ft}^2$	$\text{kg m}^2$	slugs $\text{ft}^2$	$\text{kg m}^2$	slugs $\text{ft}^2$	$\text{kg m}^2$
7969	10805	7974	10811	8055	10921

The empty case agrees almost exactly with the firm's original estimate for the aircraft at that weight.

The increment due to full internal fuel is smaller than the estimated value of 80 slugs  $\text{ft}^2$  (108  $\text{kg m}^2$ ) but even so would appear to be within  $\pm 1\%$  overall, which is within the experimental error.

The external fuel increment is very close to the calculated value, within  $\pm 1\%$  overall.

## 6 CONCLUSIONS

These experiments have shown the firm's estimates of the roll inertia to be in error by about 30%, but their estimates of pitch inertia and inclination of the principal inertia axis were very close to the measured values.

When the experimental values for roll inertia were applied to analyses which had previously been done using the firm's original estimate, the results for the stability and control derivatives based on flight-test measurements were found to be in much closer agreement with corresponding results derived from wind tunnel data, see Ref 8.

It is clear that the rig needs modification if the roll inertia of any larger aircraft is to be measured. Either the roll springs need to be much

stiffer, which is probably impractical as matching them gets more difficult, or the distance of the springs to the knife edge should be increased, giving a longer moment arm. This could be achieved by fitting outriggers on the roll frame to which the springs could be attached.

REFERENCES

<u>No.</u>	<u>Author</u>	<u>Title, etc</u>
1	C.S. Barnes A.A. Woodfield	Measurement of the moments and product of inertia of the Fairey Delta 2 aircraft. R & M 3620.
2	D.H. Perry	Measurement of the moments of inertia of the Avro 707B aircraft. RAE Technical Note Aero 2778 (ARC 23439 CP 647) (1961)
3	L.J. Fennel	Measurement of the Moments of Inertia of the Handley Page HP 115 aircraft. RAE Technical Report 65203 (1965)
4	R. Rose	Comparisons of measurements and estimates of moments of inertia of aircraft. RAE Technical Memorandum Aero 951 (AGARD CP 17 Pt 2) (1966)
5	A. Jean Ross G.W. Foster T. Turvey	An investigation of Dutch roll and wing rock oscillations of a Gnat trainer aircraft: Flight tests and linear analysis. RAE Technical Report 78032 (1978)
6	J. Clark	Measurement of moment of inertias of a Hunter II aircraft. RAE Technical Note Aero 2615 (1959)
7	M.D. Frost	The inertial effects of springs in resource and free oscillation experiments to determine damping. AARC Rep ACA-61
8	A. Jean Ross	Corrigendum and Addendum to RAE Technical Report 78032 (Ref 5 above). Technical Report to be published

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Fig 1

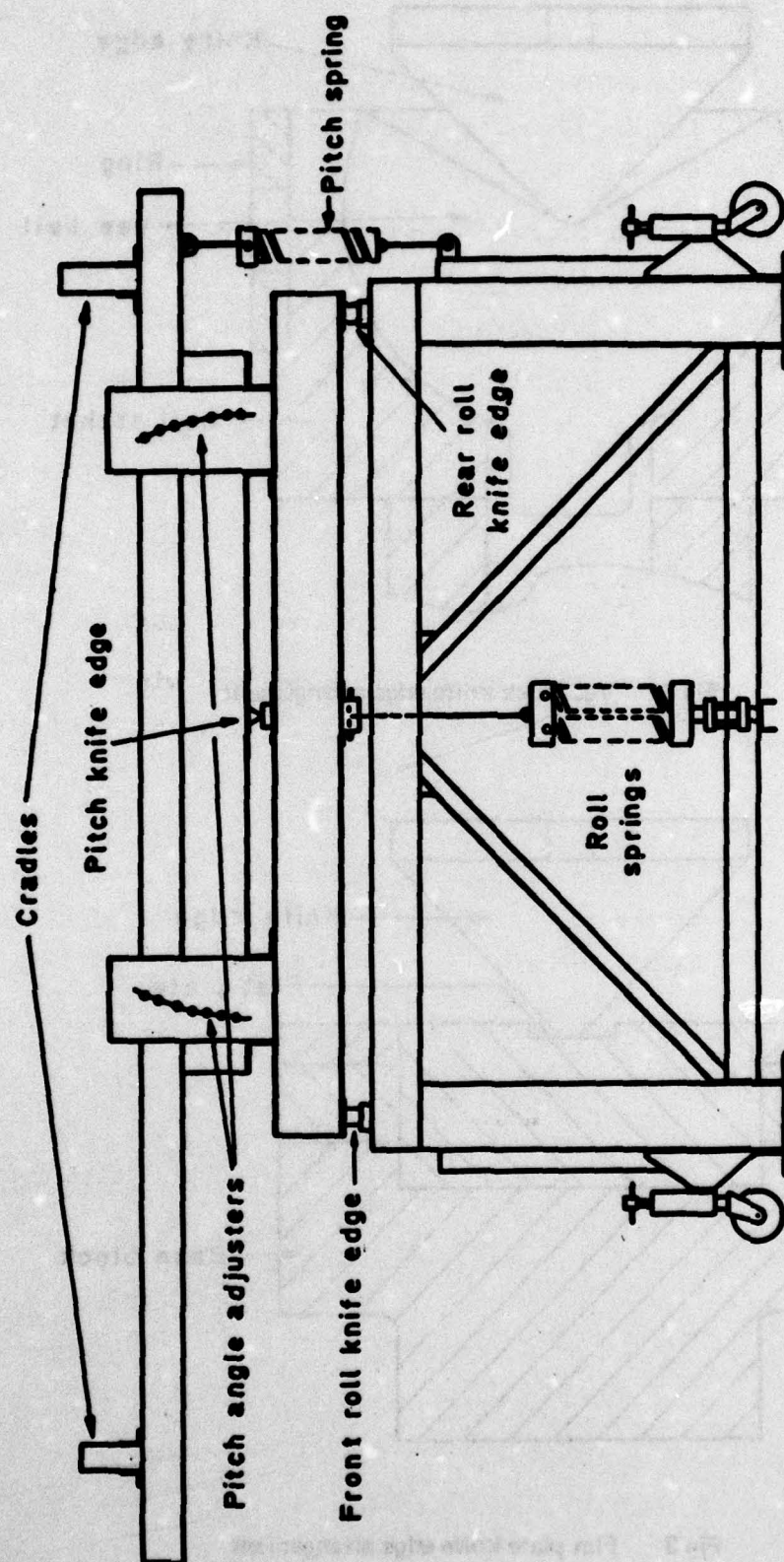
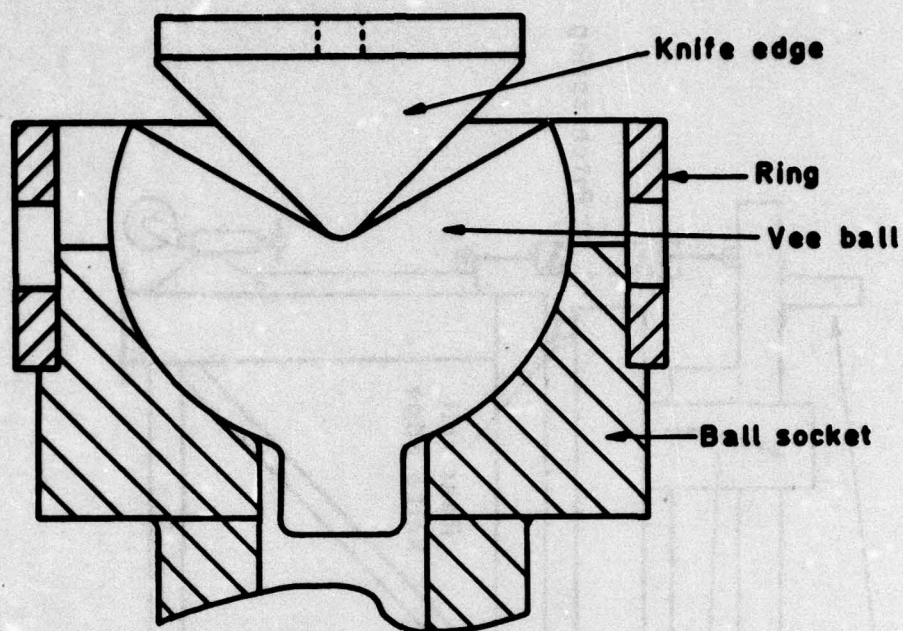
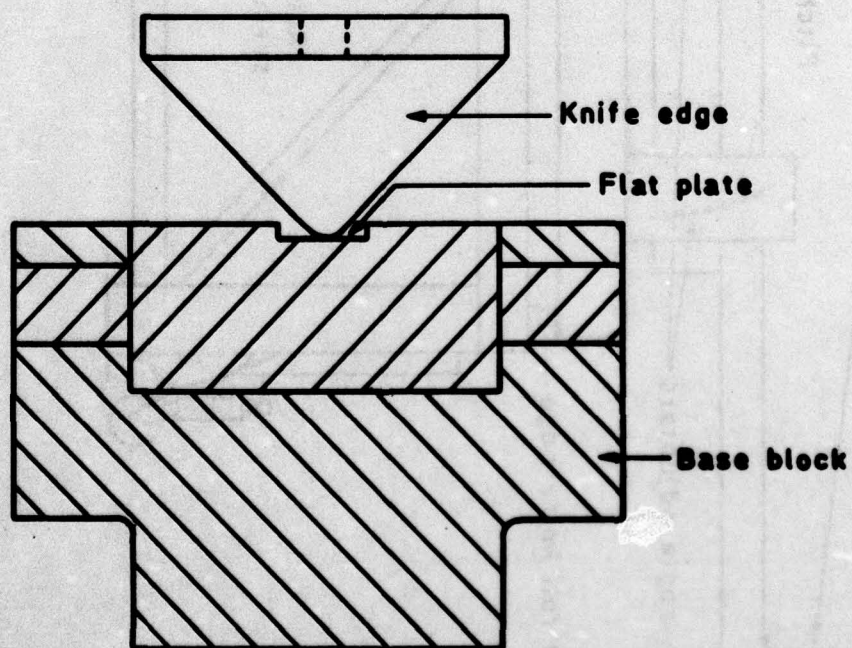


Fig 1 Elevation of inertia rig

**Figs 2&3**



**Fig 2 Vee block knife edge arrangement**



**Fig 3 Flat plate knife edge arrangement**

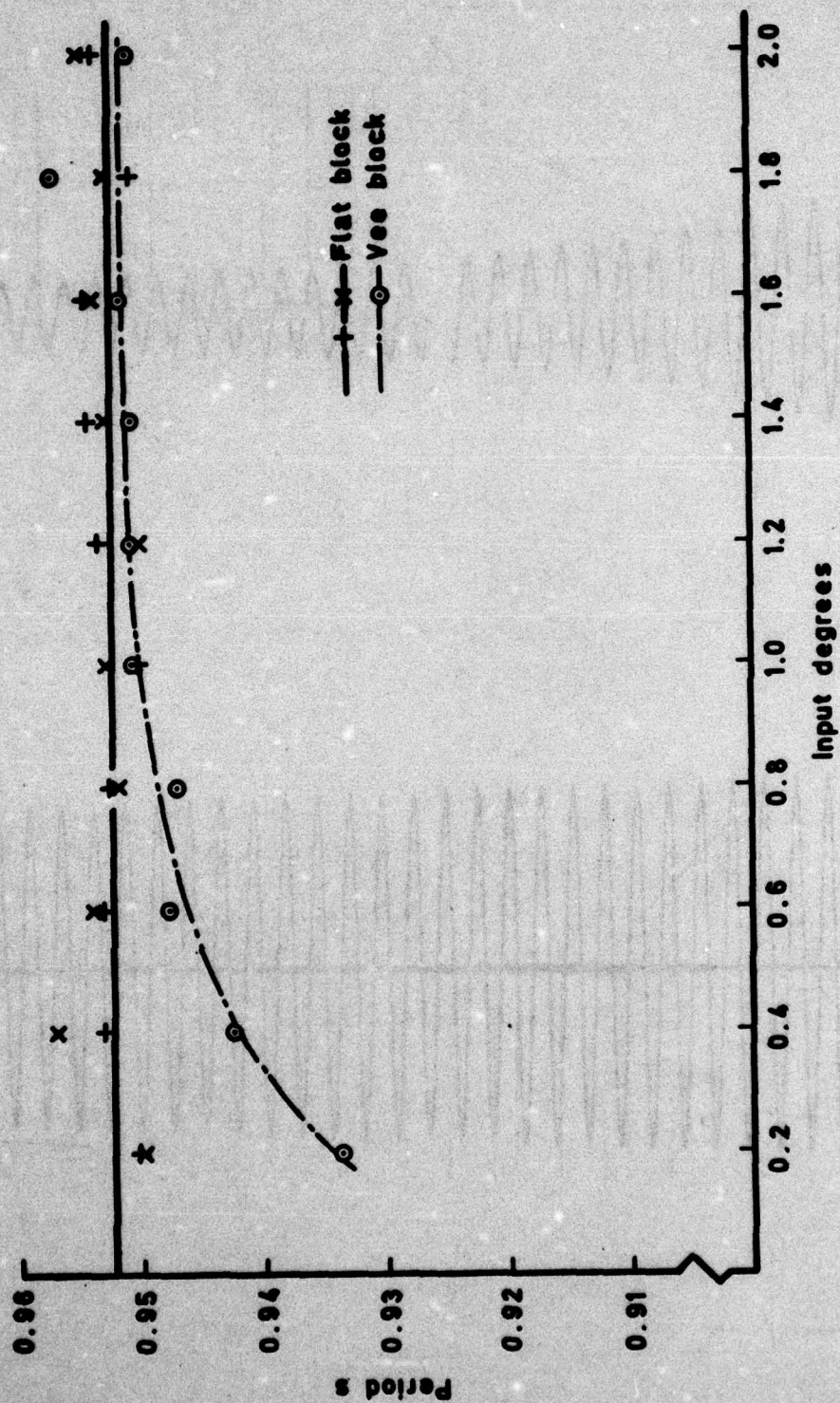
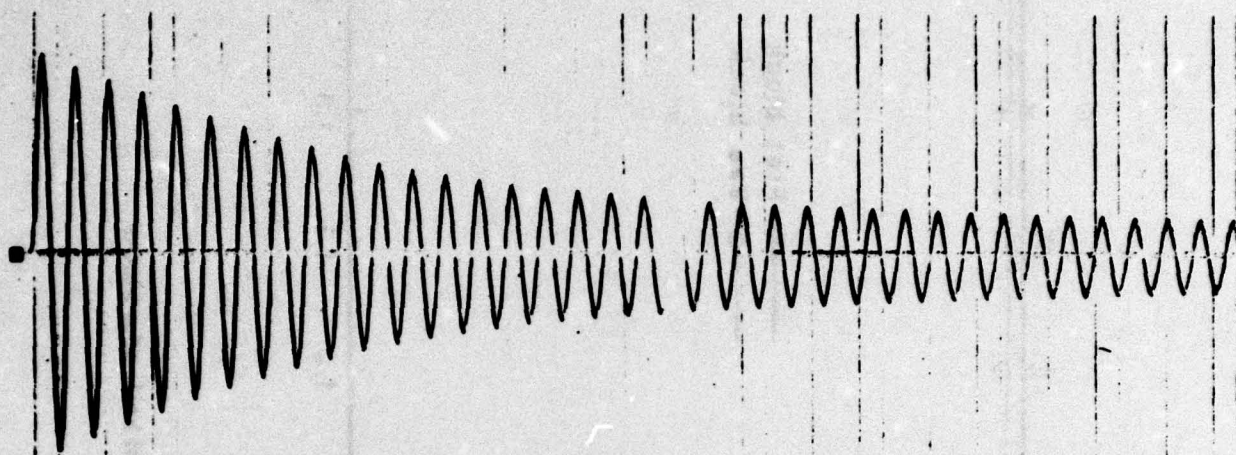
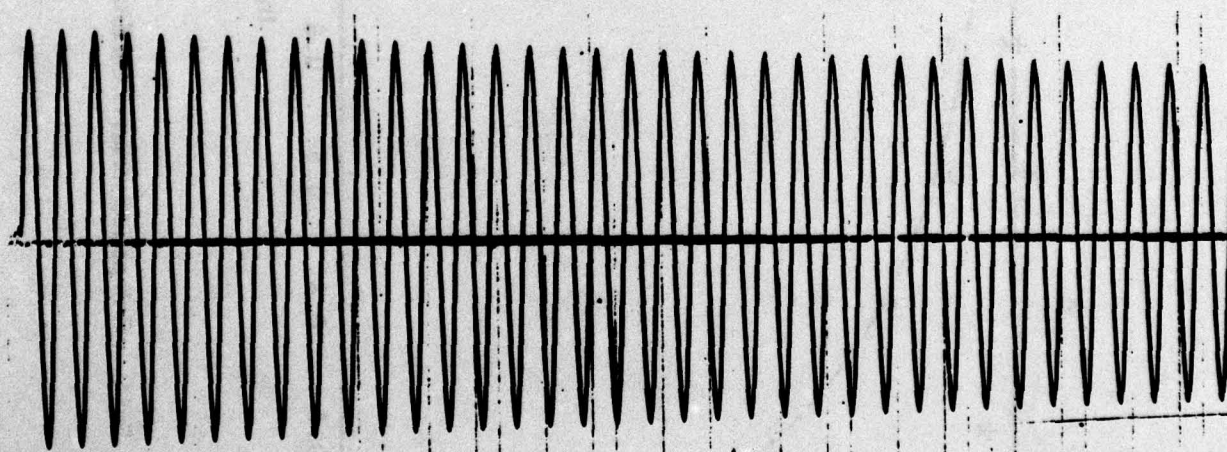


Fig 4 Roll period versus initial displacement for different knife edge blocks

**Fig 5**



**Vee blocks**



**Flat plates**

**Fig 5** Comparison of the motions using vee blocks and flat plates for the same inputs

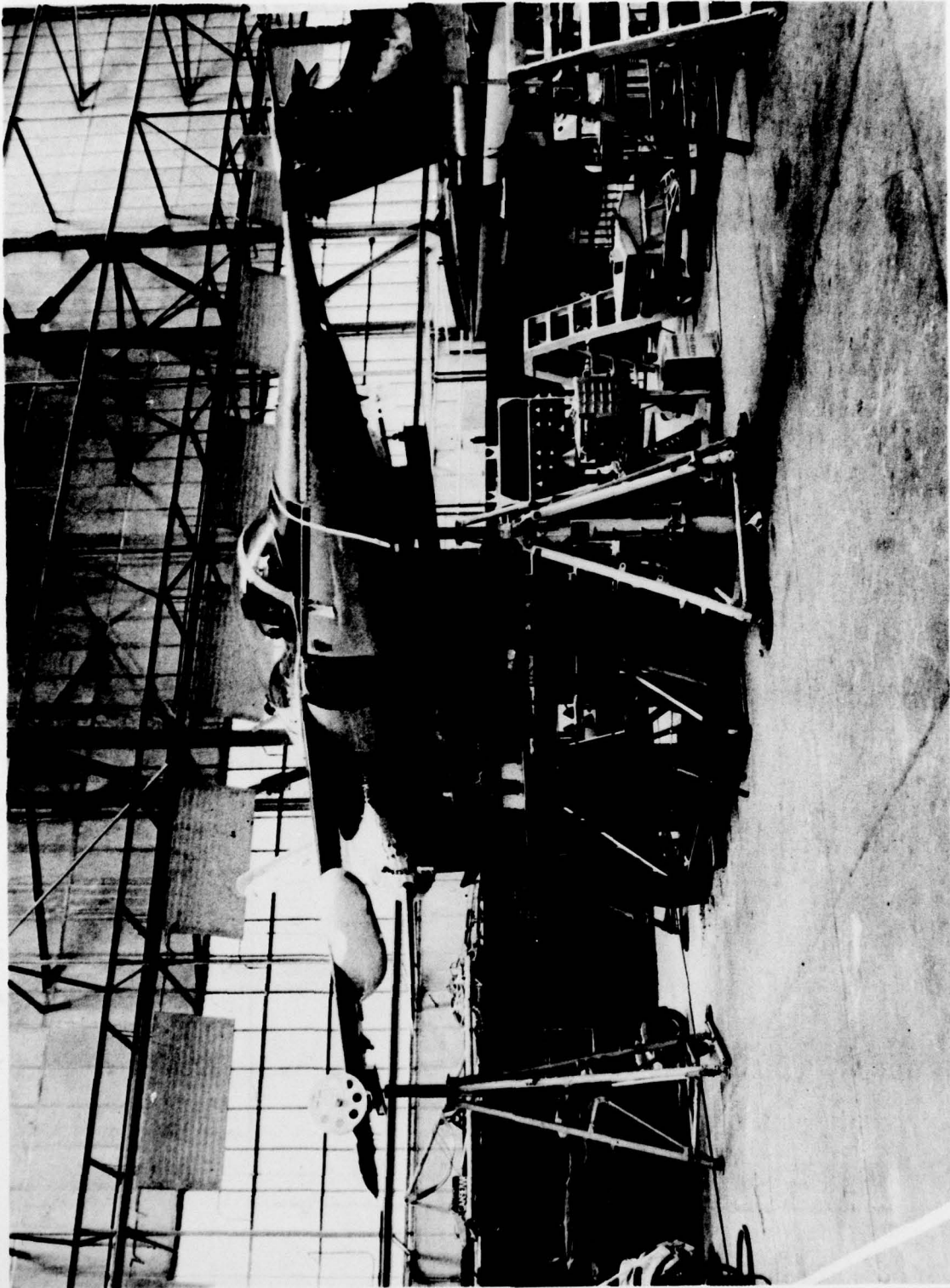


Fig 6

Fig 6 The aircraft on the inertia rig

Fig 7

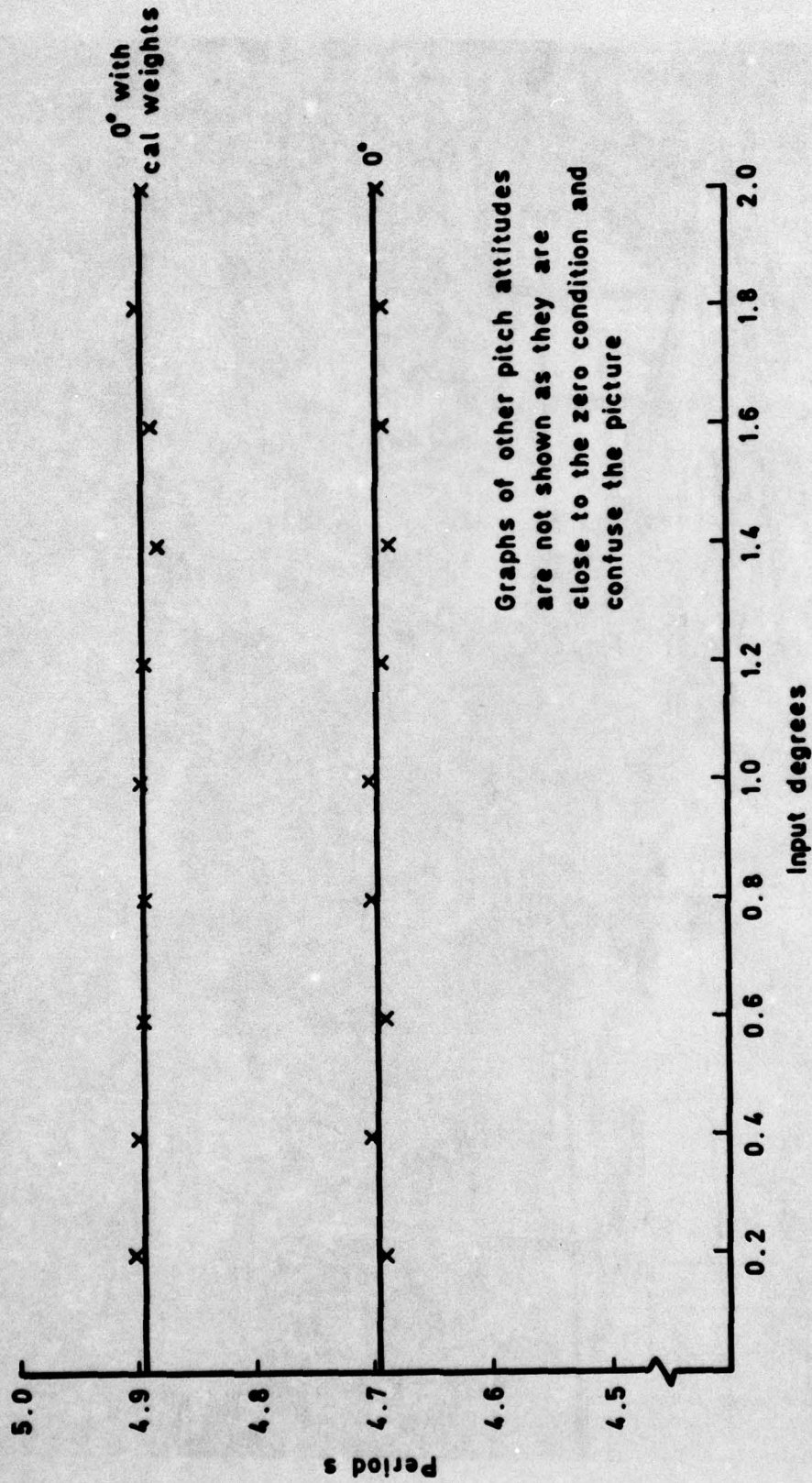


Fig 7 Roll period versus amplitude (no fuel)

Fig 8

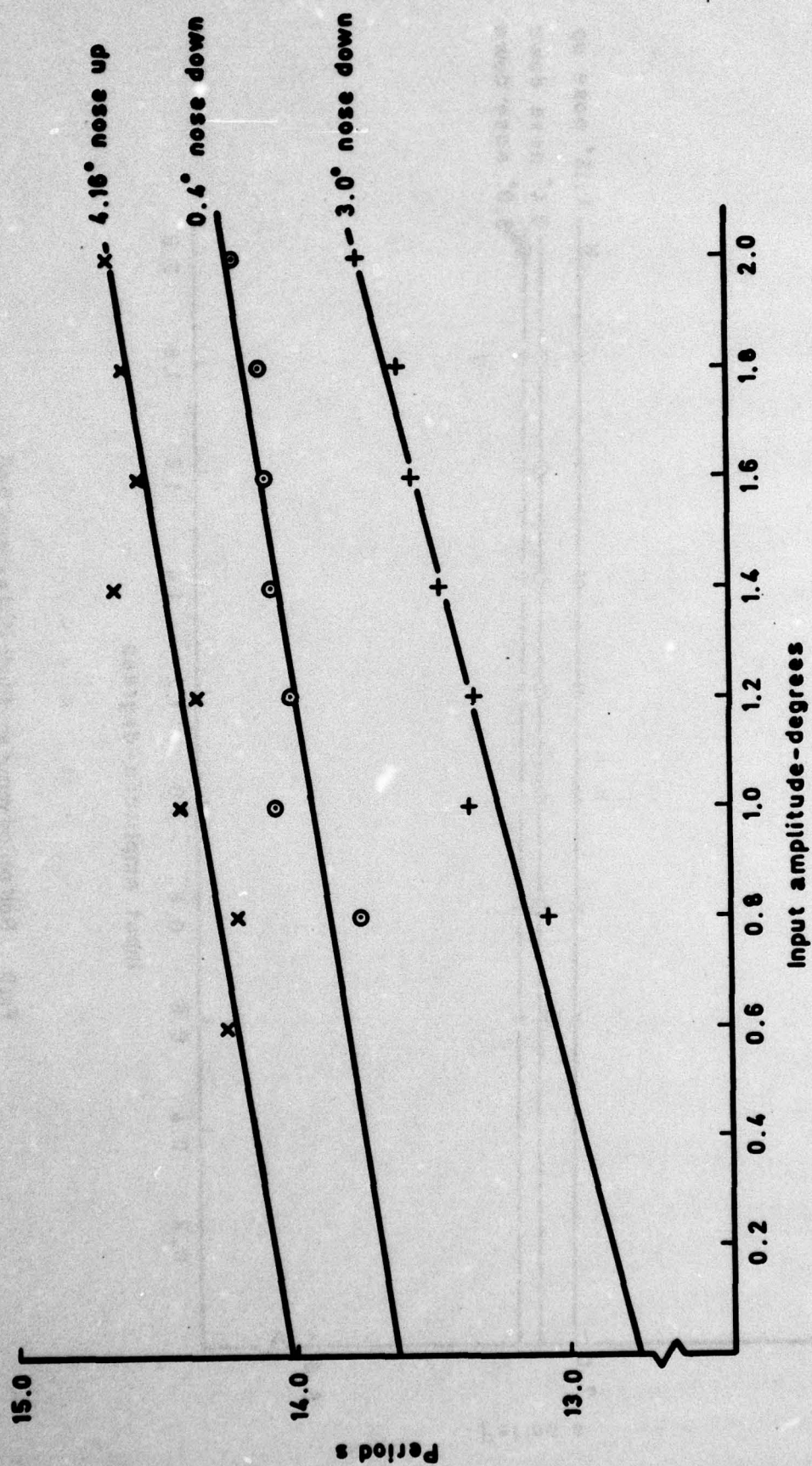


Fig 8 Roll period versus amplitude (full internal fuel)

Fig 9

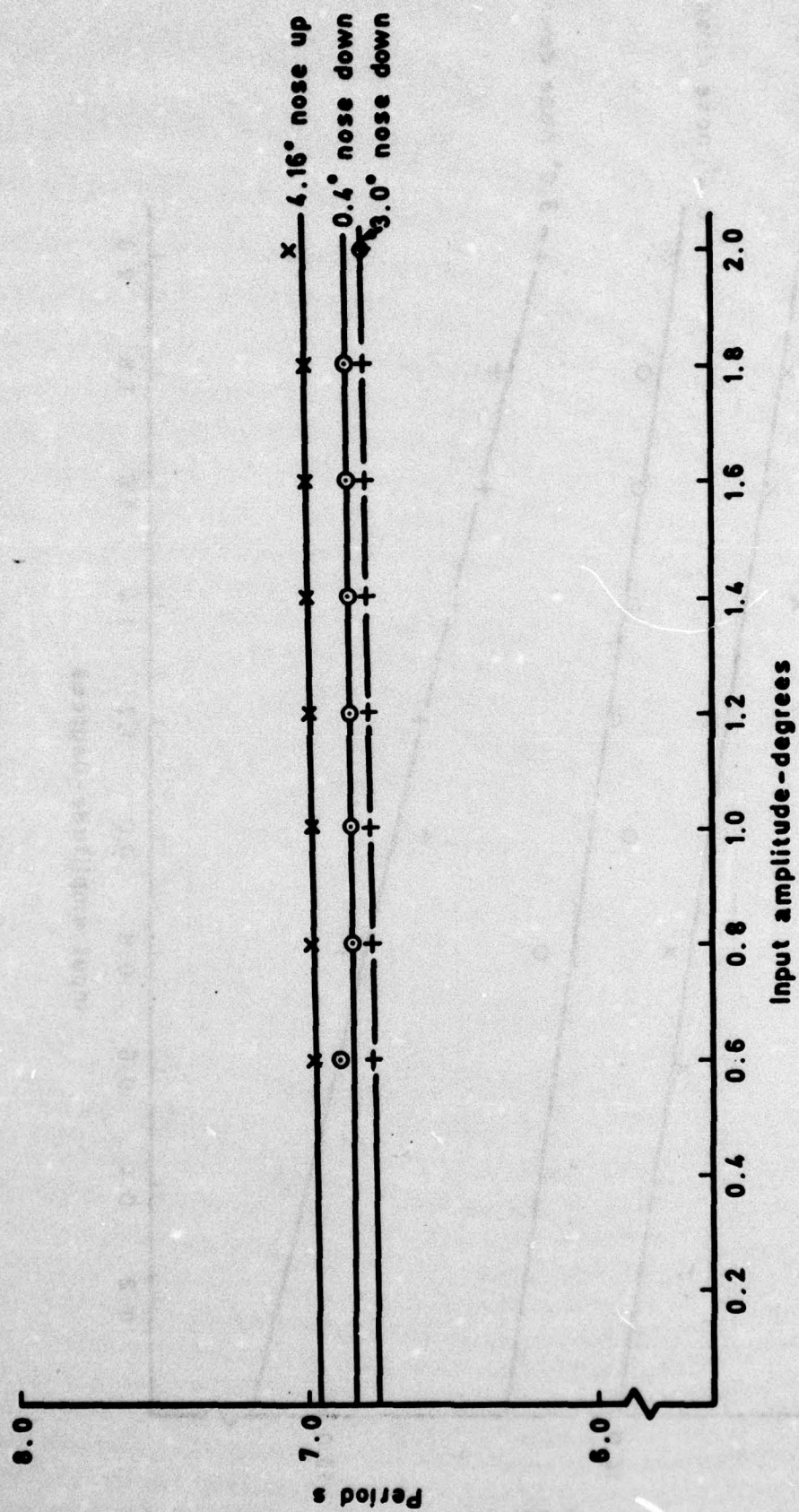


Fig 9 Roll period versus amplitude (full external fuel)

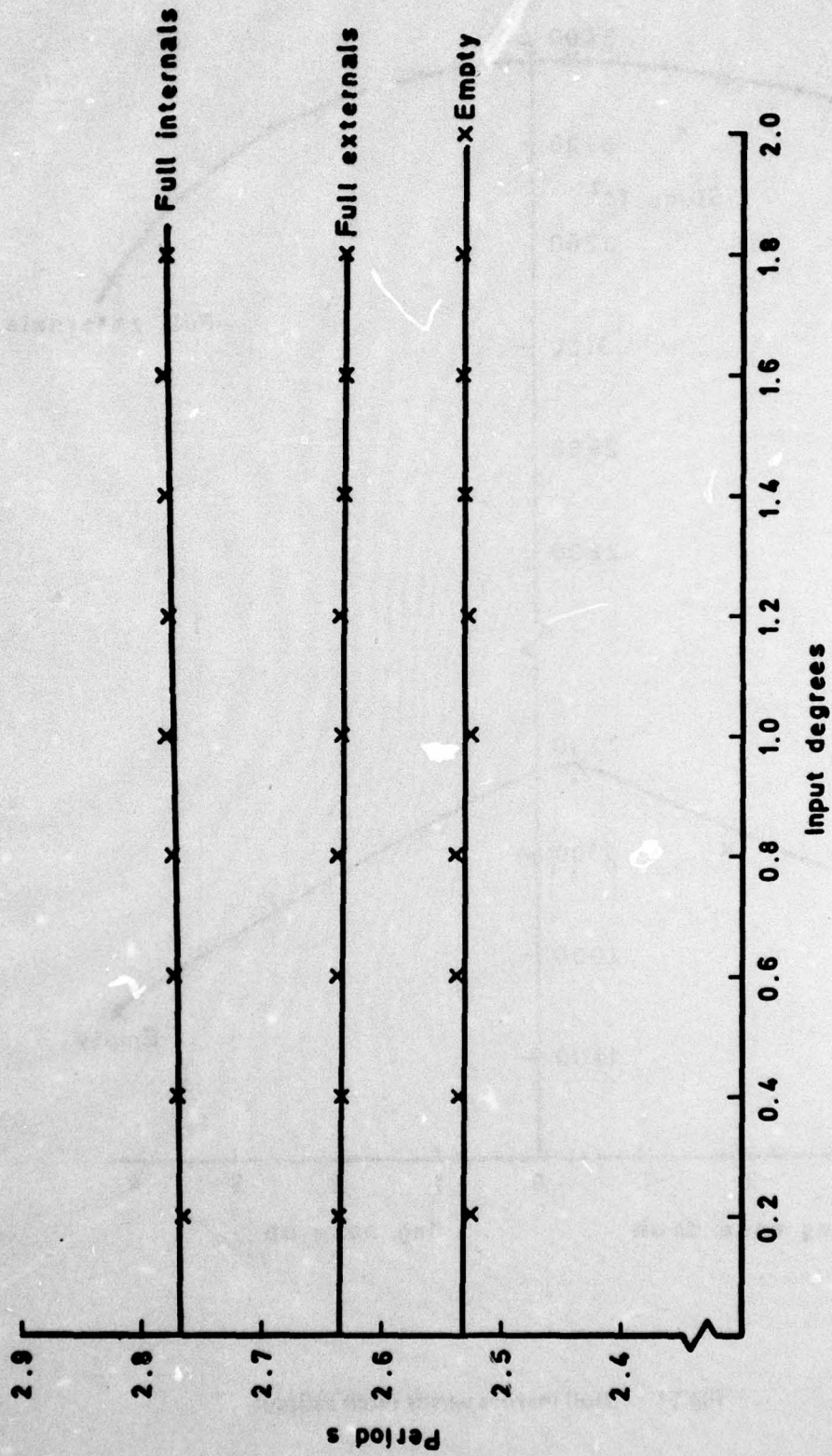


Fig 10 Pitch period versus amplitude

Fig 11

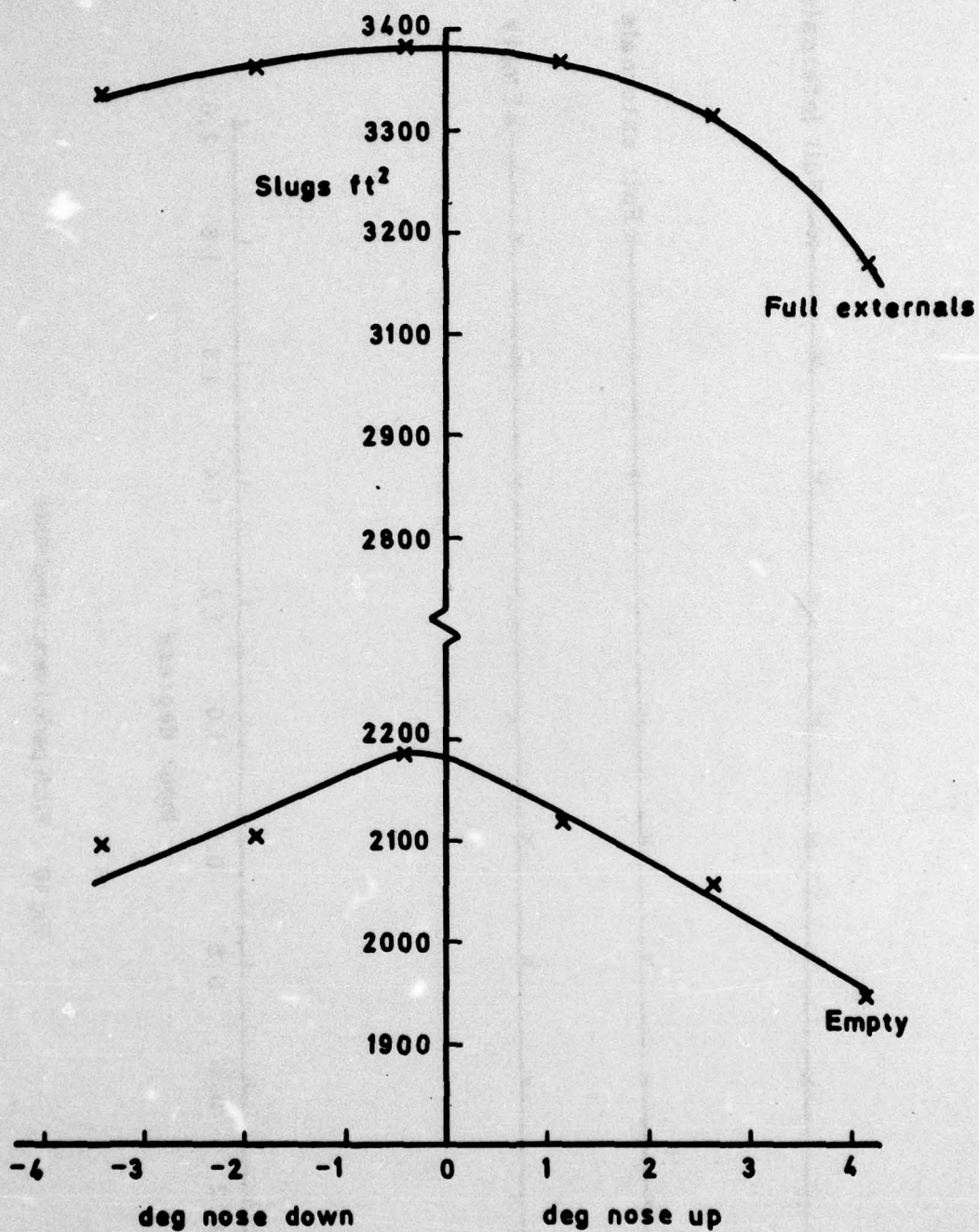


Fig 11 Roll inertia versus pitch attitude

Fig 12

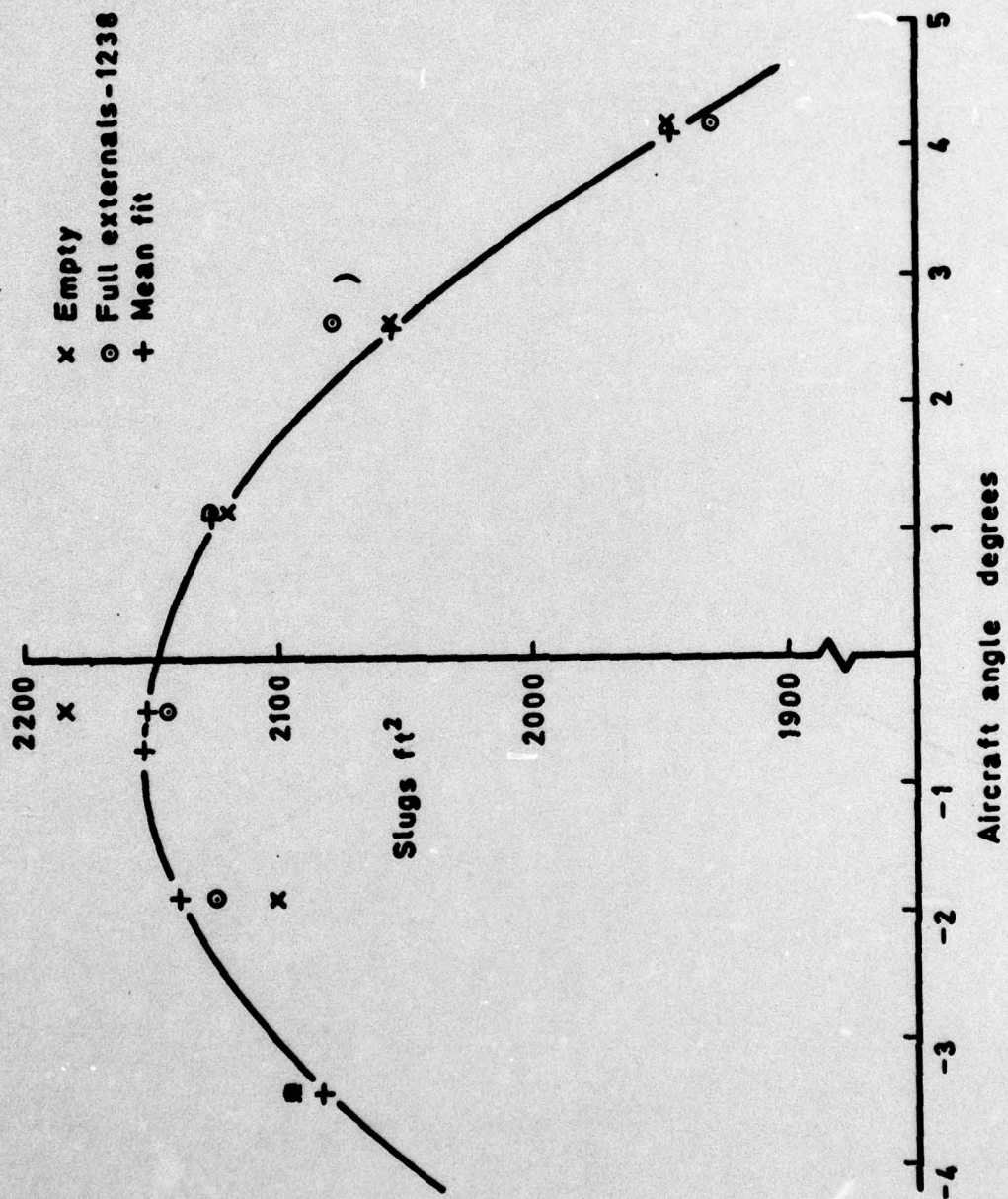


Fig 12 Roll inertia versus pitch attitude mean fit (empty)

# REPORT DOCUMENTATION PAGE

Overall security classification of this page

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17. Abstract The modifications made to a ground rig used to measure moments of inertia, and the technique developed to minimise errors in the moment of inertia in roll, are described. Calibration of the rig shows that acceptable accuracies are obtained, and results for the moments of inertia in roll and pitch, and the inclination of the principal inertia axis of the Gnat aircraft are given. Three fuel states, empty, external tanks full and internal tanks full, were tested, and comparisons are made with estimated values where possible.					